MATH 540

Numerical Computing

Project #1

Wonsang Cho

In this project you will look at solving the Allen-Cahn equation

is a user specified parameter.

1. Write a Julia module for solving (1) which supports:
2. Sparse finite difference discretizations;
3. Spectral Galerkin discretizations;
4. Periodic and Neumann boundary conditions;
5. Backwards Euler and Crank-Nicolson time stepping.

It should also support user defined values for

1. The domain ;
2. the parameter;
3. determining the mesh;
4. the time step, and the number of steps;

Within a Jupyter notebook, demonstrate that calling these functions for construction, assembly, and running the problem work correctly.

1. The uniform state, is an exact solution to this with both periodic boundary conditions and homogeneous boundary conditions. Build a test unit for your module for all combinations of spatial discretizations and time stepping and boundary conditions. For the test unit use and . The discretization at the end should be, within floating point, .

You should report your test results in a Jupyter notebook with the code:

Graphical user interface, text, application

Description automatically generated

in a Jupyter cell.

1. Focusing on the case of periodic boundary conditions, with and assess the convergence in space and time for the problem, checking both spatial discretizations and both time stepping schemes. Choose and (independently) to compare against a high resolution solution for the initial condition .

What convergence rates do you see in the different cases? Are these what you would anticipate?

Visualize the results in your Jupyter notebook.

1. Picking reasonable values of and , with and , integrate . Visualize the result on a space-time plot using contour.

Your results should be submitted with a single Jupyter notebook and a link to the GitHub repository in which you developed the code. A portion of your grade will be based on seeing that you effectively made use of git during development.